Math 8850: Intro to Quasi-Categories

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Class Meetings: MW 5:00-6:15pm

Office hours: Fri 3:00-5:00pm or by appointment. *Please feel free to talk with me outside of office hours if these are inconvenient!*

Course text: Lecture notes on course webpage. For additional references, see the end of the syllabus.

Course Overview: In the past fifty years, various versions of higher categories have become common in many branches of mathematics. Fukaya categories in symplectic geometry, derived/higher stacks in algebraic geometry, and $\mathbb{E}_n$-algebras in homotopy theory (to name just three examples) all require some kind of higher-categorical "coherent" language to formulate and work with. As higher-categorical methods have proliferated, Joyal's theory of *quasi-categories* has enjoyed particularly rapid development and great popularity in recent decades, in large part due to the work of Lurie in extending this theory and connecting it to algebraic geometry.

While this course will concern the 'technology' of quasi-categories rather than applications thereof, the importance of this technology rests on such applications. To whet your appetite, here is a sampling of applications of higher category theory:

- The equivalence between (formal) moduli problems and differential graded Lie algebras in characteristic zero was proven (separately) by Lurie and by Pridham, both using higher-categorical techniques.
- The fact that Factorization Homology/Topological Chiral Homology defines a fully extended topological field theory was proven using higher-categorical techniques by Scheimbauer.
- Weibel's conjecture on the K-groups of schemes was proven using higher-categorical techniques by Kerz, Strunk, and Tamme.
- And many more…

The aim of this course is to provide an introduction to the theory of quasi-categories, up to a point where students will be able to access the burgeoning literature making use of quasi-categorical techniques. While a number of monographs on quasi-category theory exist, these are often highly technical and short on intuitive explanation. This course will therefore aim at providing both rigor and intuition for the fundamentals of quasi-category theory.
Among other topics, this course will cover:

- Some basics on model categories.
- Simplicial sets as models for spaces and ∞-categories.
- The Kan-Quillen, Joyal, and Bergner model structures, and their relations to one another.
- Basic constructions in quasi-category theory.

Potential further topics include (depending on pace, student background, and interest):

- The Grothendieck construction.
- Monoidal ∞-categories.
- ∞-operads.
- Stable ∞-categories.

Prerequisites

Some background knowledge will be necessary to follow the course.

- You should have completed Algebraic Topology I prior to the beginning of this course.
- You should be familiar with the basic constructions of category theory: categories, functors, natural transformations, limits, colimits, Kan extensions, and adjunctions. (See Background literature)

Taking Algebraic Topology II either prior to or at the same time as this course could be desirable, but is not necessary. We will occasionally use more advanced facts from algebraic topology as black-boxes during the course.

Mental Health & Wellbeing

University study can be stressful, and the material in this course will likely be challenging for many of you. You should always feel free to contact the me, both with questions about the material and with any other concerns about the course.

The University of Virginia offers a number of helpful resources for students. Psychological counseling for students is provided by Counseling and Pyschological services (CAPS). Alternatively, there is the anonymous HELP line run by Madison House.

Evaluation & Grading

There will be weekly exercise sheets, and students will be expected to hand in solutions to at least 60% of the exercises assigned. The final grades for the course will be decided based on one-on-one discussions of the material at the end of the semester.

Accommodations

Students with a disability which requires accommodation should contact the Student Disabilities Access Center (SDAC). Students without accommodation letters from the SDAC will not be provided accommodations in class or on exams.

Course literature

The primary reference for the course will be a set of typed lecture notes I will provide. However, the following works are relevant, and may be useful references during the course.

- Bergner, J. 2018: *The Homotopy Theory of* $(\infty, 1)$*-Categories.* Cambridge University Press. 284 pp.
- Cisinski, D.-C. 2020: *Higher Categories and Homotopical Algebra.* Cambridge University Press, 446 pp. (Available online)
- Goerss, P. and Jardine, R. 1999: *Simplicial Homotopy Theory.* Birkhäuser, 510 pp.
- Groth, M. 2015: *A short course on* ∞*-categories.* 77 pp. (arXiv:1007.2925)
- Hovey, M. 1999: *Model Categories.* American Mathematical Society. 209pp.
- Joyal, A. 2008: *Notes on Quasi-Categories.* 244 pp. (Available online)
- Lurie, J. 2009: *Higher Topos Theory.* Princeton University Press. 944 pp. (Available online)
- Riehl, E. 2014: *Categorical Homotopy Theory.* Cambridge University Press. 292 pp. (Available online)

Background literature

If your categorical background is a bit shaky, I would recommend perusing one or both of the following texts as a refresher. The former is, in my opinion, slightly better reading, and contains an amazing wealth of examples.

- Riehl, E. 2016: *Category Theory in Context.* Dover. 258 pp. (Available online)
- Leinster, T. 2014; *Basic Category Theory.* Cambridge University Press. 191 pp. arXiv:1612.09375

Academic Honesty

You are encouraged to work together on the exercises for this course. Feel free to consult any resources you need to better understand the material.

Instructor Communication

Resources for this course will be posted on the course website. Throughout the semester, I may send out emails with additional information, including the scheduling of examinations, canceled or rescheduled classes, and information about exercises. You can communicate with me via email or by wandering into my office.

Meet your instructor

This is my third semester as a postdoc at UVA. Before coming here, I worked as a postdoc at Universität Hamburg in Germany. I completed my doctoral studies in 2019 at Universität Bonn, also in Germany. I specialize in higher category theory — a branch of mathematics sometimes referred to as "generalized abstract nonsense."