# Homework 9

DUE: Wednesday, 10. Nov. 2021

**Exercise 1.** Let $n > 1$, and $0 \leq i \leq n$. Show that (in $\mathsf{Grpd}$)

$$\operatorname*{colim}_{\Delta^k \to \Lambda^n_i} E(k) \cong E(n)$$

where $E(k)$ is the cosimplicial object from Homework 4, exercise 3.

**Exercise 2.** We call a morphism of simplicial sets $f : X \to Y$ a *inner fibration* if $f$ has the right lifting property with respect to every *inner* horn inclusion $\Lambda^n_i \to \Delta^n$. Show that, for any functor of 1-categories $F : \mathsf{C} \to \mathsf{D}$, the nerve

$$N(F) : N(\mathsf{C}) \to N(\mathsf{D})$$

is an inner fibration.

**Definition.** Recall the definition of a 2-category from Homework 1. Denote by $\mathsf{Cat}_2$ the category whose objects are (small) 2-categories, and whose morphisms are strict 2-functors. Denote by $\mathbb{\Delta}^n$ the 2-category with objects $0, 1, \ldots, n$, and such that

- For every $i \leq j$, there is a morphism $\phi_{i,j} : i \to j$ such that $\phi_{i,i} = \mathrm{id}_i$.
- For every $i = i_0 \leq i_1 \leq \cdots \leq i_k = j$, there is a unique 2-morphism

$$\phi_{i_{k-1},j} \circ \cdots \circ \phi_{i_1,i_2} \circ \phi_{i_0,i_1} \Rightarrow \phi_{i,j}$$

**Exercise 3.** Show that the assignment

$$\begin{aligned} D : \Delta &\longrightarrow \mathsf{Cat}_2 \\ [n] &\longmapsto \mathbb{\Delta}^n \end{aligned}$$

defines a functor. For a 2-category $\mathbb{C}$, let $N_2(\mathbb{C})$ be the simplicial set defined by

$$N_2(\mathbb{C})_n := \mathrm{Hom}_{\mathsf{Cat}_2}(D(n), \mathbb{C}).$$

What information is needed to specify a 2-simplex in $N_2(\mathbb{C})$? What information is necessary to specify a 3-simplex?

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