Math 8850, Fall 2021
Walker H. Stern

Homework 8

Due: Wednesday, 3. Nov. 2021

Exercise 1. Let $(\mathsf{C}, \mathcal{C}\mathrm{of}_{\mathsf{C}}, \mathcal{F}\mathrm{ib}_{\mathsf{C}}, \mathcal{W}_{\mathsf{C}})$ be a model category, and $f : X \to Y$ a morphism in C. Fix fibrant-cofibrant replacements

$$X \underset{\sim}{\overset{p_X}{\twoheadleftarrow}} QX \underset{\sim}{\overset{i_X}{\hookrightarrow}} RQX$$

$$Y \underset{\sim}{\overset{p_Y}{\twoheadleftarrow}} QY \underset{\sim}{\overset{i_Y}{\hookrightarrow}} RQY$$

Complete the argument shown in class to show that morphisms $RQf : RQX \to RQY$ making the

$$\begin{array}{ccccc}
X & \underset{\sim}{\overset{p_X}{\twoheadleftarrow}} & QX & \underset{\sim}{\overset{i_X}{\hookrightarrow}} & RQX \\
{\scriptstyle f}\downarrow & & {\scriptstyle Qf}\downarrow & & \downarrow{\scriptstyle RQf} \\
Y & \underset{\sim}{\overset{p_Y}{\twoheadleftarrow}} & QY & \underset{\sim}{\overset{i_Y}{\hookrightarrow}} & RQY
\end{array}$$

commute are unique up to homotopy. More precisely, show that the equivalence class $[RQf]$ only depends on f.

Exercise 2. Let $(\mathsf{C}, \mathcal{C}\mathrm{of}_{\mathsf{C}}, \mathcal{F}\mathrm{ib}_{\mathsf{C}}, \mathcal{W}_{\mathsf{C}})$ and $(\mathsf{D}, \mathcal{C}\mathrm{of}_{\mathsf{D}}, \mathcal{F}\mathrm{ib}_{\mathsf{D}}, \mathcal{W}_{\mathsf{D}})$ be model categories. Let $F : \mathsf{C} \to \mathsf{D}$ be a functor which preserves cofibrations and trivial cofibrations, i.e. such that $F(\mathcal{C}\mathrm{of}_{\mathsf{C}}) \subset \mathcal{C}\mathrm{of}_{\mathsf{D}}$, and $F(\mathcal{C}\mathrm{of}_{\mathsf{C}} \cap \mathcal{W}_{\mathsf{C}}) \subset \mathcal{C}\mathrm{of}_{\mathsf{D}} \cap \mathcal{W}_{\mathsf{D}}$. Show that if $f : X \to Y$ is a weak equivalence between cofibrant objects in C, then $F(f) \in \mathcal{W}_{\mathsf{D}}$.

Exercise 3. Let $(\mathsf{C}, \mathcal{C}\mathrm{of}_{\mathsf{C}}, \mathcal{F}\mathrm{ib}_{\mathsf{C}}, \mathcal{W}_{\mathsf{C}})$ be a model category, and let $X \in \mathsf{C}$ be an object. Denote by $\mathsf{C}_{/X}$ the slice category. Show that there is a model category structure on $\mathsf{C}_{/X}$ whose fibrations, cofibrations, and weak equivalences are inherited from C.