Math 8850, Fall 2021
Walker H. Stern

Homework 7

Due: Wednesday, 27. Oct. 2021

Exercise 1. Let $\mathcal{M}$ denote the collection of all monomorphisms in Set_Δ (i.e., degree-wise injective maps). Show that $\mathcal{M}$ is saturated. Conclude that $\mathcal{M} = \overline{\{\partial\Delta^n \to \Delta^n\}_{n\geq 0}}$.

Exercise 2. Let $i : A \to B$, $j : C \to D$, and $p : X \to Y$ be morphisms of simplicial sets. Show that each lifting problem

$$\begin{array}{ccc} A \times D \coprod_{A\times C} B \times C & \longrightarrow & X \\ {\scriptstyle i\wedge j}\big\downarrow & & \big\downarrow{\scriptstyle p} \\ B \times D & \longrightarrow & Y \end{array}$$

uniquely corresponds to a lifting problem

$$\begin{array}{ccc} A & \longrightarrow & \mathrm{Map}(D, X) \\ \big\downarrow & & \big\downarrow \\ B & \longrightarrow & \mathrm{Map}(C, X) \times_{\mathrm{Map}(C,Y)} \mathrm{Map}(D, Y) \end{array}$$

and that the former has a solution if and only if the latter does.

Exercise 3. Let $\mathcal{D}$ and $\mathcal{N}$ be sets of morphisms in Set_Δ. Set

$$\mathcal{Q} := \{f \mid f \wedge g \in {}_\perp\mathcal{D} \text{ for all } g \in \mathcal{N}\}.$$

Show that $\mathcal{Q}$ is saturated. (Hints: (1) use exercise 1, (2) it suffices to check the case where $\mathcal{N}$ consists of a single morphism).

Exercise 4. Recall the sets of morphisms from class:

$$\mathcal{B}_2 := \left\{ \{i\} \times \Delta^n \coprod_{\{i\}\times\partial\Delta^n} \Delta^1 \times \partial\Delta^n \to \Delta^n \mid i \in \{0, 1\},\ n \geq 0. \right\}$$

$$\mathcal{B}_2 := \left\{ f \wedge g \mid {}^{f:\{i\}\to\Delta^1}_{g:A\hookrightarrow B \text{ monomorphism}} \right\}$$

Show that $\overline{B_2} = \overline{B_3}$.