Math 4720: Introduction to Differential Geometry

Fall 2022

Instructor: Walker H. Stern
Email: ws7jx (at) virginia.edu
Office: Kerchof 311

Class Meetings: MWF 12:00–12:50pm, Monroe Hall 114

Office hours: Fri 3:00-5:00pm or by appointment.

References: This course does not have a fixed textbook. Instead, I will be lecturing based on several references. I will endeavor where possible to post my lecture notes on my website each weekend. The main references the course will draw from are:

> Wilhelm Klingenberg. *A Course in Differential Geometry.* Springer Graduate Texts in Mathematics, Volume 51. ISBN: 978-1461299257
>
> Wolfgang Kühnel. *Differential Geometry: Curves — Surfaces — Manifolds.* AMS. ISBN: 978-1470423209
>
> Manfredo P. Do Carmo. *Differential Geometry of Curves and Surfaces: Revised and Updated Second Edition.* Dover Books on Mathematics. ISBN: 978-0486806990

Course format: The course will consist of in-person instruction and class sessions will not be recorded. Each student is responsible for keeping up-to-date on course material.

Course Overview: This course is intended to be an introduction to differential geometry, more particularly the differential geometry of embedded manifolds. Roughly speaking, we will study curves, surfaces, and their higher dimensional analogs in terms of notions like distance, length, and curvature, using differentiation and integration. In this context, the word *differential* refers to the fact that we will be making use of the tools and techniques of calculus. *Geometry* roughly refers to the fact that we will be using concepts like distance and curvature (this stands in contrast to the subject of differential topology where distances are not necessarily well-defined). Finally, *embedded* means we will only be considering those spaces which are subspaces of n-dimensional real Euclidean space, $\mathbb{R}^n$.

This course will be *proofs-based*, meaning that you will be expected to be able to read, write, and understand mathematical proofs. Throughout the course I will assume familiarity with basic concepts from multivariable calculus, differential equations, and linear algebra. Some material from real analysis will also be used, though for the purposes of this course some such theorems can be treated as black boxes.

The aim of the course will be to reach a pair of profound statements about the nature of curved surfaces in $\mathbb{R}^n$. The *Gauss-Bonnet Theorem* relates several notions of curvature to an invariant called the Euler characteristic. The *Theorema Egregium* — also proven by Gauss — shows that the curvature of a surface depends only on only a few types of information about that surface. Along the way to these two theorems, we will encounter concepts like Frenet frames, arc length, curvature, tangent spaces, and fundamental forms. Time permitting, we may also discuss additional topics such as Minkowski space, manifolds, differential forms, and tensors.

Evaluation & grading

Evaluation for this course has two components: Homework exercises, and a final exam. The way the final grades will be computed is somewhat unusual, and is explained below. **Do not worry about the unfamiliar grading format.** Experience has shown that it both provides more accurate evaluation of student understanding and tends to raise student grades. We will discuss the grading during the first class session. Please feel free to contact me with concerns about the course.

Homework: There will be weekly written exercise sheets, which will be handed in via Gradescope. Gradescope is available from the course Collab page. You are strongly encouraged to work together with other students on the exercises. However, you are **expected to write up individual solutions, and hand them in individually.** The individual write-ups should **not** be simply copied from one another.

Final: The final examination will be a **cumulative oral examination**, intended to be taken in the last week of the semester. To be allowed to take the final, you **must have an average score of at least 65%** on the homework assignments. (At this point, I should say **don't worry**, if you put effort into the class, it should not be hard to clear this bar.) If your homework average is below this threshhold, you will not be allowed to sit the exam.

Final retake: If you are unsatisfied with your performance, **you may elect to take the exam a second time**, during the scheduled finals period. The questions asked will be different. If you take the exam twice, **only your second score will count.**

Final grades: Final letter grades will be assigned based exclusively on your performance on the oral exam. If you do not have a sufficient homework average to be admitted to the exam, you will be counted as having failed the course.

Piazza

This course will make use of the discussion forum Piazza on Collab. I strongly encourage all of you to discuss any questions you have about the material there with your classmates and me. I will regularly check Piazza, and endeavor to respond rapidly to any questions which appear there.

Deadlines, make-up exams, and late work

As a general rule, late work will not be accepted and late assignments will be given no credit. In exceptional cases, extensions may be granted due to extenuating circumstances.

Final: Since each of you will arrange the time of your final exam with me, no conflicts should arise. In the even of a last-minute conflict or emergency, you should inform me as soon as possible, and a make-up exam will be arranged.

Mental Health & Wellbeing

University study can be stressful, and the material in this course will likely be very new for many of you. You should always feel free to contact me, both with questions about the material and with any other concerns about the course.

The University of Virginia offers a number of helpful resources for students. Psychological counseling for students is provided by Counseling and Psychological services (CAPS). Alternatively, there is the anonymous HELP line run by Madison House.

Accomodations

Students with a disability which requires accommodation should contact the Student Disabilities Access Center (SDAC). Students without accommodation letters from the SDAC will not be provided accommodations in class or on exams.

Academic honesty

All exams in this course fall under the purview of the UVA honor code. Working together on homework is permitted, and is not considered a violation of any course policy.

Instructor Communication

Throughout the semester, I will send you emails through the course Collab page. You are responsible for the contents of these communications. These communications may include the scheduling of examinations, canceled or rescheduled classes, or information about homework. You may communicate with me via email, Collab, or wandering into my office. **Note: I will primarily answer emails during normal business hours. Emails received at night, on weekends, or on holidays may go unanswered until the next work day.**

Meet your instructor:

This is my fifth semester as a postdoc at UVA. Before coming here, I worked as a postdoc at Universität Hamburg in Germany. I completed my doctoral studies in 2019 at Universität Bonn, also in Germany. I specialize in higher category theory — a branch of mathematics sometimes referred to as "generalized abstract nonsense." For those interested, my professional webpage is walkerstern.gitlab.io