BACKGROUND ON SETS

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This is going to be a brief review of some notation which we will use throughout the course.

Sets. Loosely speaking, a *set* is just a collection of things, called *elements*. A set can have a finite number of elements, or an infinite number of elements. A set can be denoted by curly braces; for example

$$\{\text{red, green, blue}\}$$

is the set containing elements 'red', 'green', and 'blue'. Most, though not all, of the sets we will use in this course will be sets of numbers. There are a few common sets of numbers, with standard notation:

$$\begin{array}{ll} \mathbb{N} & \text{the natural numbers, } \{0,1,2,\ldots\} \\ \mathbb{Z} & \text{the integers, } \{\ldots,-2,-1,0,1,2,\ldots\} \\ \mathbb{R} & \text{the real numbers} \end{array}$$

The symbol '$\in$' means 'is an element of'. So, for instance, $y \in X$ would be read as 'y is an element of (the set) X'. Similarly, $\pi \in \mathbb{R}$ would be read as 'π is an element of the set of real numbers', or, more simply, 'π is a real number'.

Logical symbols. There are a few main logical symbols we will use in this course:

$$\begin{array}{rl} \forall & \text{`For all'} \\ \exists & \text{`There exists'} \\ \Rightarrow & \text{`implies'} \end{array}$$

So, for instance we might write a 'sentence'

$$\forall x \in \mathbb{R},\ \exists y \in \mathbb{R} \quad \text{such that} \quad y < x$$

which we could read as

'For every x in the set of real numbers, there exists a y in the set of real numbers such that $y < x$.

As another example, we might write

$$(\exists n \in \mathbb{N} \text{ such that } \theta = n\pi) \Longrightarrow \sin(\theta) = 0$$

which we could read as

'If there exists a natural number n such that $\theta = n\pi$, then $\sin(\theta) = 0$.'

Building sets from other sets. We will also occasionally make use of *set-builder notation*. This is a way of quickly writing down a specific set. It again uses curly braces to define a set, but now divides the curly braces into two sections with a line '$|$'. The symbols before the line tell us what elements of the set will look like, and the symbols after the line tell us what conditions the elements have to satisfy.

For example we could write

$$\{x \in \mathbb{R} \mid x < 0\}$$

which would be translated as 'The set of real numbers x such that $x < 0$', or, more simply, 'The set of negative real numbers'.

We can also write conditions in words, for instance, we might define

$$\mathbb{P} := \{n \in \mathbb{N} \mid n \text{ is prime}\}$$

i.e. the line above defines $\mathbb{P}$ as 'The set of all natural numbers which are prime'.

There is one other way which we will build sets from other sets: *products of sets.* If we have a set X and a set Y, we can define a set

$$X \times Y := \{(x, y) \mid x \in X \text{ and } y \in Y\}$$

So, translating to words, $X \times Y$ is the set of ordered pairs (x, y) where x is an element of the set X, and y is an element of the set Y.

One particularly common product we will consider is $\mathbb{R} \times \mathbb{R}$, the set of ordered pairs of real numbers. We think of this as the plane, since the coordinates of points in the plane are simply ordered pairs of real numbers. We will usually write $\mathbb{R} \times \mathbb{R}$ as $\mathbb{R}^2$. Similarly, we think of the product $\mathbb{R} \times \mathbb{R} \times \mathbb{R}$ as 3-dimensional space — ordered triples of real coordinates, rather than ordered pairs — and we write it as $\mathbb{R}^3$